Eighth Annual Upper Peninsula High School Math Challenge

Northern Michigan University (Marquette MI, USA)

Saturday April 8, 2017

RELAY 1 Category: AREA PLAYER 1

A circle of radius 3 inches and a circle of radius 4 inches are concentric (they share a center). What, in square inches, is the area of the annulus (ring) between the two circles?

Pass your answer to PLAYER 2.

AREA OF OUTER CIRCLE IS $\pi \cdot 4^2$. AREA OF INNER CIRCLE IS $\pi \cdot 3^2$. AREA BETWEEN IS $16\pi - 9\pi = \boxed{7\pi}$.

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RELAY 1 Category: FACTORING PLAYER 2

The number you will receive from PLAYER 1 should be of the form $n \cdot \pi$, where n is an integer. What is the **smallest** positive integer that has **exactly** n factors?

Pass your answer to PLAYER 3.

PLAYER 1: 7π $n = 7$

A number with exactly 7 factors must be a prime raised to the sixth power. There is no other way to do it. The smallest is $2^6 = \boxed{64}$.

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RELAY 1 Category: MEAN, MEDIAN, MODE PLAYER 3

Four basketball players have a mean height, in inches, equal to the number you will receive from PLAYER 2. The height, in inches, of three of the players are 60, 61, and 65. What is the median height of the players in inches?

Pass your answer to PLAYER 4.

PLAYER 2: 64

x = height of fourth player

$$\frac{x+60+61+65}{4} = 64 \qquad x + 186 = 256$$

$$x = 70$$

60, 61, 65, 70. Median is halfway between 61 and 65, so $\boxed{63}$.

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RELAY 1 Category: QUADRATIC EQUATIONS PLAYER 4

Let n be the number you will receive from PLAYER 3. Which root of the equation $x^2+16x+n = 0$ has the largest absolute value?

Run your answer to the front.

PLAYER 3: 63

$x^2+16x+63=0$

$(x+7)(x+9)=0$

$x=-7 \quad x=-9$

$\boxed{-9}$ has largest absolute value.

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RELAY 2 Category: PALINDROMES PLAYER 1

What is the **smallest** palindrome (number that reads the same forwards and backwards) whose digits add up to 17?

Pass your answer to PLAYER 2.

Make the middle digit as large as possible so that first digit is as small as possible.

$\boxed{494}$

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RELAY 2 Category: COIN PROBLEM PLAYER 2

The number you will receive from PLAYER 1 is the number of cents I have in nickels and pennies. I have 102 coins total. How many of them are pennies?

Pass your answer to PLAYER 3.

PLAYER 1 : 494

$n+p = 102$

$5n+5p = 510$

$5n+p = 494$

$4p = 16$

$p = \boxed{4}$

$5n+p = 494$

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RELAY 2 Category: POLYGONS PLAYER 3

The number you will receive from PLAYER 2 is the number of sides of a regular polygon. If the length of each side is 1.9 inches long, what, in square inches, is the area of this polygon?

Pass your answer to PLAYER 4.

PLAYER 2: 4

Shape is a square. $1.9^2 = 3.61$

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RELAY 2 Category: INFINITE SERIES PLAYER 4

Take the number you receive from PLAYER 3 and round it off to the nearest integer. This number is the first term of an infinite geometric series. Every term in the series is $\frac{2}{3}$ of the preceding term. What is the sum of this series?

Run your answer to the front.

PLAYER 3: 3.61 rounds to 4,

sum of an infinite geometric series: $S = \frac{a}{1-r}$

$S = \frac{4}{1-\frac{2}{3}} = \frac{4}{\frac{1}{3}} = 4 \cdot 3 = \boxed{12}$

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RELAY 3 Category: SOLID GEOMETRY PLAYER 1

An ice cream cone is made by taking a hemisphere of radius 1 inch and attaching to it a cone of radius 1 inch and height 3 inches. In cubic inches, what is the volume of the shape given by the entire ice cream cone?

Pass your answer to PLAYER 2.

$V_{hemisphere} = \frac{2}{3}\pi r^3 = \frac{2}{3}\pi$

$V_{cone} = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \cdot 3 = \pi$

Total: $\frac{2}{3}\pi + \pi = \boxed{\frac{5}{3}\pi}$

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RELAY 3 Category: TRIGONOMETRY PLAYER 2

The number you will receive from PLAYER 1 is of the form $r \cdot \pi$ where r is a positive rational number. Suppose that r is the secant of a Quadrant I angle. What is the tangent of that angle?

Pass your answer to PLAYER 3.

PLAYER 1: $\frac{5}{3}\pi$ $r = \frac{5}{3}$

$\tan^2\theta + 1 = \sec^2\theta$ $\tan^2\theta + 1 = \frac{25}{9}$ $\tan^2\theta = \frac{16}{9}$ $\tan\theta = \boxed{\frac{4}{3}}$

(positive since Quadrant I)

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RELAY 3 Category: WORK PROBLEM PLAYER 3

Let p/q be the reduced, improper fraction that represents the number you receive from PLAYER 2. If you can eat p hot dogs in a minute, and I can eat q, and we start at the same time, how many hours until you have eaten 100 more hot dogs than I have?

Pass your answer to PLAYER 4.

PLAYER 2: $4/3$

$p = 4 \quad q = 3$

Each minute, you eat 1 more hot dog than I do.

It will take 100 minutes to eat 100 extra hot dogs.

100 min = 1 hr 40 min or $\boxed{1\frac{2}{3}}$ hr

In that time, you have eaten 400 hot dogs, you pig!

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RELAY 3 Category: AGE PROBLEM PLAYER 4

Let p/q be the reduced, improper fraction that represents the number you receive from PLAYER 3. In p years, George will be q times the age that Sue is now. If George is presently q years older than Sue, how old is George now?

Run your answer to the front.

PLAYER 3: $1\frac{2}{3}$ or $\frac{5}{3}$

$p = 5 \quad q = 3$

~~$g + p = 3q$~~

~~$g + p = 3q$~~ $\qquad g = q + 5$

~~$g + 5 = g$~~

$g + p = qs \qquad g = 3 + s$

$g + 5 = 3s \qquad s = g - 3$

$g + 5 = 3(g - 3)$

$g + 5 = 3g - 9$

$2g = 14$

$g = \boxed{7}$